An MDP Model of Vehicle-Pedestrian Interaction at an Unsignalized Intersection

Ya-Chuan Hsu
Dept. of Computer Science and Computer Engineering
Texas A&M University
Email: yachuan815@tamu.edu

Swaminathan Gopalswamy
Dept. of Mechanical Engineering
Texas A&M University
Email: sgopalswamy@tamu.edu

Srikanth Saripalli
Dept. of Mechanical Engineering
Texas A&M University
Email: ssaripalli@tamu.edu

Dylan A. Shell
Dept. of Computer Science and Computer Engineering
Texas A&M University
Email: dshell@tamu.edu

***Abstract*—Though autonomous vehicles are currently operating in several places, many important questions within the field of autonomous vehicle research remain to be addressed satisfactorily. In this paper, we examine the role of communication between pedestrians and autonomous vehicles at unsignalized intersections. The nature of interaction between pedestrians and autonomous vehicles remains mostly in the realm of speculation currently. Of course, pedestrian's reactions towards autonomous vehicles will gradually change over time owing to habituation, but it is clear that this topic requires urgent and ongoing study, not least of all because engineers require some working model for pedestrian-autonomous-vehicle communication. Our paper proposes a decision-theoretic model that expresses the interaction between a pedestrian and a vehicle. The model considers the interaction between a pedestrian and a vehicle as expressed an MDP, based on prior work conducted by psychologists examining similar experimental conditions. We describe this model and our simulation study of behavior it exhibits. The preliminary results on evaluating the behavior of the autonomous vehicle are promising and we believe it can help reduce the data needed to develop fuller models.**

I. Introduction

This paper focuses on communication between an autonomous vehicle and a pedestrian at an unsignalized intersection. This circumstance arises in crossings that span a road and intersections without traffic lights and stop signs.

At unsignalized intersections, pedestrians communicate their intention to drivers by gesturing or making eye contact. Drivers might acknowledge by waving a hand back or returning the eye contact. This communication establishes an agreement on who has the priority to cross. When the vehicle is an autonomous vehicle, and there is no driver, to whom will pedestrians direct their communication and what signals would encode affirming signals in return?

Patterns of communication between autonomous vehicles and pedestrians are not yet not clearly established. People have started to conduct research in how people communicate with autonomous vehicle [1] and begun to develop different ways to structure the reciprocal interaction. For example, Lagström and Lundgren [2] added LED lights to an autonomous vehicle to help display the vehicle's intentions. Understanding of a pedestrian reactions remains limited and one anticipates that it will evolve as autonomous vehicles and their associated technologies become ubiquitous.

This paper presents some preliminary work in developing a model that expresses the driver-pedestrian interaction at an intersection without traffic signals. Simulating the model allows one to examine different pedestrian crossing behaviors and, crucially, their coupling with a vehicle. We believe it may be useful for psychologists, for example, in informing their subsequent studies.

II. Literature Review

A. Pedestrian Crossing Behavior

A review of the literature indicates that several factors contribute to pedestrian crossing behavior, chief among them are environmental factors, social factors, cultural factors, and dynamic factors. Environmental factors include traffic signals, zebra crossings, the width of the crosswalk, etc. For example, pedestrians are more conservative at non-designated crossings and unsignalized zebra crossings. At a signalized crossing, pedestrians have been shown to cross while neglecting how the vehicle acting (e.g. speeding up). This is interpreted as pedestrian's having faith in the driver following the law and hence ultimately stopping at the signal [3].

Social factors are also shown to have a great impact on pedestrian crossing behavior. One of the factors is the size of the pedestrian group. Group size determines the degree to which pedestrians obey the law [4]. Another factor is age: the elderly are more conservative while crossing [5], most children are, in general, inadequately attentive to the road [6].

The influence a vehicle has on a pedestrian's crossing behavior contributes to so-called 'dynamic factors'. The vehicle's speed and its position relative to the crossing point are considered by the pedestrian when a crossing decision is made. Time-gap acceptance is commonly identified when dynamic factors are studied. The pedestrian's time-gap acceptance also varies across differences in road settings, age, gender, country, etc. [7].

Though there are multiple factors that pedestrians consider when crossing, a general crossing pattern can still be distilled. Gorrini, Vizzari, and Bandini [8] state that, based on different deceleration/acceleration trends, crossing behaviors can be divided into three sequential phases: approaching, appraising and crossing. Rasouli, Kotseruba, and Tsotsos [3] categorize crossing actions based on their meaning in terms of actions: the precondition to crossing, attention, reaction to driver's actions and crossing itself.

B. Driver Behaviour at Crossings

Reciprocally, driver behaviour at crossings is also influenced by multiple aspects. These aspects include the group size of the crossing pedestrians, the distance of the pedestrian(s) to the crosswalk, and the size of the city [9]. Additionally, the current velocity of the vehicle affects the driver's decisions when approaching a crosswalk. If the vehicle is moving at a high speed, drivers break significantly earlier [10]. An explanation is offered by the fact that drivers decelerate at a rate not exceeding $3.048\,\mathrm{m/s^2}$ for reasons of comfort [11]. The layout of the environment is another aspect that affects driver behavior. At mid-road crosswalks with a curb extension, the vehicle has an average deceleration of $-1.92\,\mathrm{m/s^2}$. This is greater than an average deceleration rate of $-2.39\,\mathrm{m/s^2}$ occurring at mid-road crosswalks with advanced yield marks, but without curb extensions [12].

C. Modeling Driver-Pedestrian Interaction

Autonomous vehicles need to consider the motion of surrounding agents to navigate safely and reliably in the complex-real world environment. Though pedestrians are only one set of these, they play a significant role and their actions can be especially challenging to predict accurately.

Schroeder [11] evaluates the vehicle-pedestrian interaction by conducting event-based data collection. Besides collecting and analyzing data, they also used logistic regression techniques to predict the vehicle pedestrian's dynamics. From a dynamic profile of both pedestrian and vehicle, they conclude that pedestrian characteristics and other events influence the crossing behavior of both pedestrian and vehicle.

Tirthankar et al. [13] incorporated intention-awareness into their approach for motion planning for autonomous vehicles. They predict a pedestrian's trajectory by maintaining a belief state of the pedestrian's goal, i.e., distribution over locations where the pedestrian may be heading. They then adapt the vehicle's action accordingly. Instead of completely recognising all pedestrians' intentions, they resolve the intentions when needed, balancing actions which gather information for resolving intention against actions to complete the navigation task.

Several studies have been conducted to observe how pedestrians and vehicles interact. However, the relationship between these studies and autonomous vehicles—and, in particular, how to gain insight that will aid the engineer in implementing a controller—remains limited. In this paper, we propose a model, aiming to bridge data and actual implementation on a robot. Our current model focuses on capturing the implicit communication mediated via changes in speed between the vehicle and the pedestrian.

III. Background

Our goal is to model the interaction between a vehicle and a pedestrian. However, from the perspective of implementing an autonomous vehicle, controls can only be sent to the vehicle; even if some actions (such as honking a horn/hooter) influence the pedestrian, they do so only indirectly. We choose to represent this as a sort of uncertain dynamics. We pose the question of which actions should be chosen (by the vehicle) and use a Markov Decision Process (MDP) to model action selection in uncertain environments.

A. MDP Preliminaries

Formally, an MDP model [14] is defined by a tuple (S, A, T, R, γ), where S and A denote the model's state space and action space respectively. The transition function T is a conditional probability function $T(s, a, s') = p(s'|s, a)$ yielding the probability of transitioning from current state $s \in S$ to the next state $s' \in S$ when taking action $a \in A$. The process is decision-theoretic, being based on the rewards R, which describe the utility of particular circumstances, and which direct a maximizing agent to choosing desirable actions. Immediate rewards are specified for each action taken in each state. The solution for an MDP, called a policy, $\pi : S \to A$ prescribes an action $a \in A$ for each state $s \in S$. More specifically, at each time step, a system in state s performing action a will receive reward $R(s, a)$. The goal of the system is to choose a policy that will maximize the accumulated reward. We calculate the maximum accumulated reward with a discounted sum over a potentially infinite horizon $\sum_{t=0}^{\infty} \gamma^t R_{a_t}(s_t, s_{t+1})$, where $\gamma \in (0, 1)$ is a discount factor that models a preference for immediate rewards over future ones.

IV. Markov Decision Process Model

In this paper, our goal is to create a simulated autonomous vehicle capable of mimicking the dynamic behavior of a human controlled vehicle, including moderating its speed in a manner reflective of the presence of pedestrians at an unsignalized intersections. We model the autonomous vehicle as an MDP agent that needs to avoid collision with an approaching pedestrian who may opt to cross the intersection.

A. Basic Dynamic Model

First, we examine how a vehicle and a pedestrian can be modeled in a decoupled manner. Then the section which follows details how they are coupled together.

We express the dynamics of a pedestrian with a Markov chain. States represent discretized distances from the intersection. The velocity of the pedestrian determines the transition probabilities between the states. To calculate the transition probabilities, we first initialize some pedestrian with a given velocity and simulate the pedestrian moving forwards at this velocity. After several rounds of simulation, we average the times the pedestrian is located in each state and transform these numbers into probabilities. In this way, we create a unique Markov chain for every pedestrian velocity that would be used in our simulation (see Fig. 1).

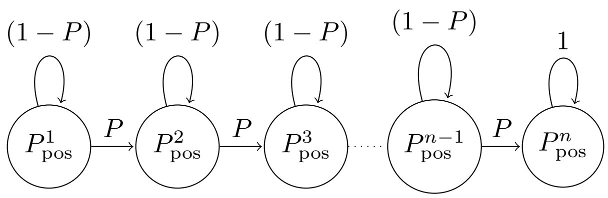

Fig. 1. Pedestrian Dynamics Markov Chain.

The vehicle's dynamics are expressed with the same method. But the states are the discretized distances from the vehicle to the intersection instead (see Fig. 2).

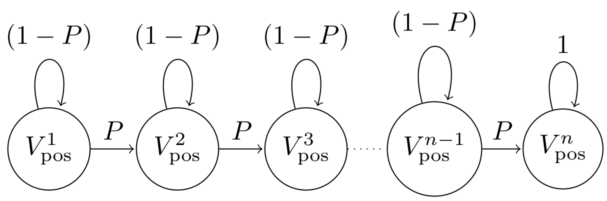

Fig. 2. Vehicle Dynamics Markov Chain.

B. MDP Model

We set the state space S of our MDP model as the joint dynamic state spaces of the vehicle and pedestrian. The set S is specified as a product: $(P_{\text{pos}}, P_{\text{vel}}, V_{\text{pos}})$. The pedestrian distance from the intersection is P_{pos}. When P_{pos} has a negative value, it represents a pedestrian that has not reached the crosswalk. The element P_{vel} represents the pedestrian speed in the direction of motion towards to the intersection. Lastly, the state space S contains the vehicle's position V_{pos}, where V_{pos} is the vehicle's distance from the intersection. A negative value represents the fact that the vehicle has not yet traversed the crosswalk. (See Fig. 3) The states characterize variables that, though coarse, express details important for the interaction between the vehicle and the pedestrian.

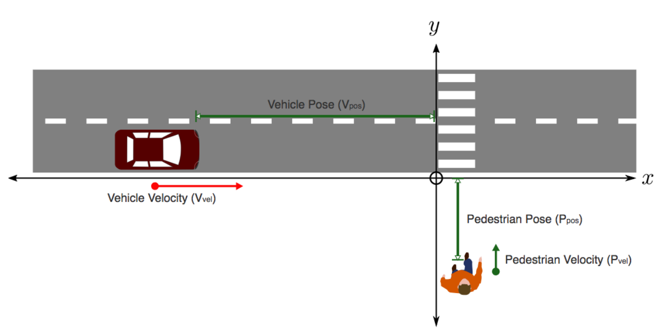

Fig. 3. God's eye view of the scene the MDP model describes.

The action space A encodes the control parameters of the autonomous vehicle. It is a set of different velocities V_{vel} that the vehicle can be commanded to drive at.

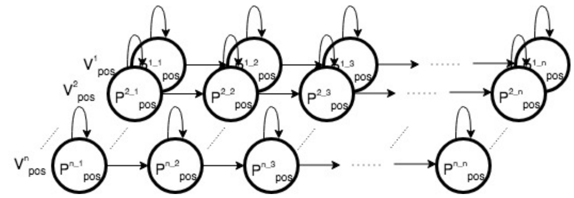

Fig. 4. MDP State Transition Diagram: For each V_{pos}, there is a Markov chain that expresses the dynamics of a pedestrian moving in speed P_{vel}. The influence of the vehicle on the pedestrian is shown by the added transition between pedestrian Markov chains.

C. Reward Model

By adopting a decision-theoretic model, we are constructing an autonomous vehicle that acts as a rational agent to minimize its risk of collision with the pedestrian (subject to the correctness of the model). Furthermore, we assume the autonomous vehicle wishes to cross the intersection as efficiently as possible. To model such behaviour within the MDP, we use the additive reward function $R(s, a) = R_{\text{col}}(s) + R_{\text{eff}}(a)$, where $R_{\text{col}}(s)$ is the penalty imposed if at state $s \in S$, the autonomous vehicle and the pedestrian is both on the crosswalk and $R_{\text{eff}}(a)$ is the cost for the autonomous vehicle to perform action $a \in A$ from state $s \in S$.

V. The Simulator

Given the problem modeled in Section IV, the motion strategy for the autonomous vehicle is generated by solving the associated MDP. Our software simulates the behavior of an autonomous vehicle and pedestrian which may interact at an unsignalized intersection.

The simulation deals predominantly with the scenario in Fig. 3. It requires two sources of input: a means by which the vehicle's speed is determined, and the same for the pedestrian. These sources, in general, can include prerecorded sequences, keyboard input, or other means. For the results we present, the vehicle's speed is always determined from a policy π^* that results from solving the MDP; determining the next action for the vehicle is then just a problem of evaluating $\pi^* [(P_{\text{pos}}(t), P_{\text{vel}}(t), V_{\text{pos}}(t))]$. This requires that the pedestrian position, pedestrian speed, and vehicle position be mapped to the equivalence class of discrete states that describe that configuration:

$$\textbf{State}^{\textbf{Sim}}(t) \mapsto (P_{\text{pos}}(t), P_{\text{vel}}(t), V_{\text{pos}}(t)).$$

The pedestrian's speed is determined in different way. A separate model is provided for different classes of pedestrian. The model is varied with the experiments, so they will be described in detail next. It is important to note that we are interested in behavior where the vehicle has an imperfect or inaccurate model. Thus, though evaluation uses a model of a pedestrian to generate that simulated pedestrian's action, the model provided as part of the MDP may differ markedly.

VI. EXPERIMENTS

We consider two different choices of pedestrian characteristics; each of these results in a different dynamics for the pedestrian. Because the MDP expresses a coupling of the pedestrian and vehicle dynamics, each of these choices leads to a particular MDP.

Having solved each MDP, we then use different pedestrian behaviors to analyze the vehicle's reaction generated by the optimal policy. In all cases, of course, the vehicle is expected to perform a series of actions to cross safely and efficiently.

A. Setting Parameters

The average velocity of a walking pedestrian is $1.4\,\mathrm{m/s}$ [15] and the fastest a pedestrian walks is $2.5\,\mathrm{m/s}$ [16]. We constructed Markov chains for pedestrians traveling at speed $0.0\,\mathrm{m/s}$ and the speeds between $1.4\,\mathrm{m/s}$ and $2.5\,\mathrm{m/s}$. For the vehicle, we set up three action choices, which are the vehicle traveling at a high speed of $10.0\,\mathrm{m/s}$, a slow speed of $5.0\,\mathrm{m/s}$, and a stop with the vehicle's velocity at $0.0\,\mathrm{m/s}$.

The design of the MDP, via a state space expressing the joint state of the vehicle and pedestrian, can capture important elements of the interaction between the two. We consider a state $(P_{\text{pos}}, P_{\text{vel}}, V_{\text{pos}})$, where P_{pos} is a set from -6 to 6 with intervals of 0.5, V_{pos} is a set from -15 to 11 with intervals of 1.5, and P_{vel} is defined differently according to the pedestrian characteristic.

1) Suicidal Pedestrian Crossing Behavior: Suicidal pedestrian crossing behavior is defined as a pedestrian that always aims to cross the intersection before the vehicle crosses. When the pedestrian reaches 2 meters from the intersection, it predicts the vehicle's arrival time according to the current vehicle's position and speed. The vehicle's position information is included in the current state space and the vehicle's speed is given by the action state the vehicle choose to perform. If the vehicle appears to be arriving at the intersection before the pedestrian finishes crossing, the pedestrian will speed up in order to cross first. The action of speeding up may cause a collision, but the pedestrian does this nevertheless. This class of crossing behavior is seen on university campuses.

This description is shown quantitatively as:

$$P_{\text{vel}}(x) = \begin{cases} 1.4 & \text{if } x > 2 \\ 2.5 \times e^{-0.289x} & \text{if } x \leq 2 \end{cases}, \quad (1)$$

where x is the time difference between the remaining time for the vehicle to arrive at the intersection and the remaining time for the pedestrian to finish crossing the intersection.

2) Cautious Pedestrian Crossing Behavior: Cautious pedestrians stop at the curb and wait for the vehicle to cross first when they determine that, continuing at their current speed, they cannot reach the other side of the road before the vehicle arrives.

Their velocity is given as:

$$P_{\text{vel}}(d) = \begin{cases} 1.4 & \text{if } d < -1 \text{ and } d > 1 \\ 0 & \text{if } -1 \leq d \leq 1 \text{ and can't cross at } 1.4\,\mathrm{m/s} \\ 1.4 & \text{if } -1 \leq d \leq 1 \text{ and can cross at } 1.4\,\mathrm{m/s} \end{cases}, \quad (2)$$

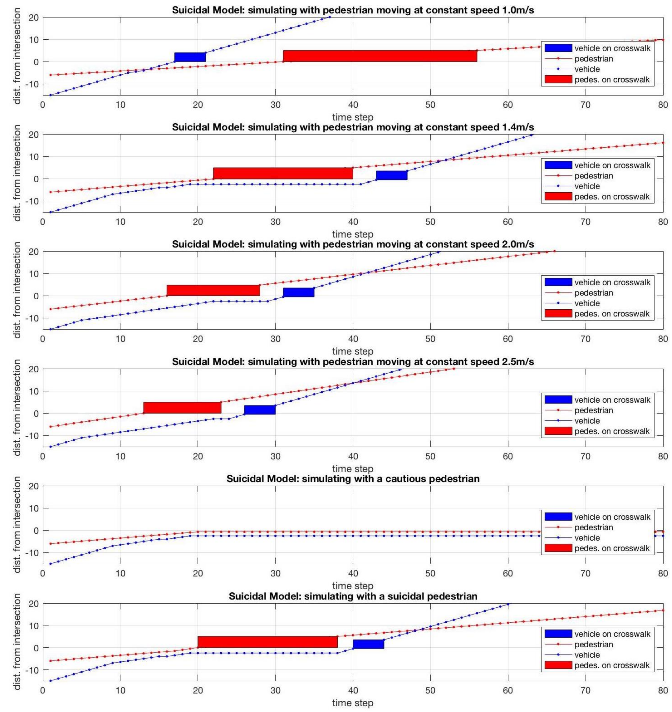

Fig. 5. Suicidal pedestrian-based MDP model: We construct a suicidal pedestrian-based MDP model and examine the autonomous vehicle's behavior by simulating the model with pedestrians moving at constant speed $1.0\,\mathrm{m/s}$, $1.4\,\mathrm{m/s}$, $2.0\,\mathrm{m/s}$ and $2.5\,\mathrm{m/s}$, a cautious pedestrian and a suicidal pedestrian as separate experimental conditions.

where d is the distance between the pedestrian and crosswalk.

Using the reward model, $R(s, a) = R_{\text{col}}(s) + R_{\text{eff}}(a)$, described in Section IV-C, we define the collision zone to be the entire crosswalk area, assigning the $R_{\text{col}}(s)$ value in the states that cover these areas a reward of -1000. $R_{\text{eff}}(a)$ is set to encourage the autonomous vehicle to finish crossing the intersection as soon as possible; $R_{\text{eff}}(a)$ is set to 10 when the vehicle action a is travel in high speed, 5 when traveling in slow speed, and -10 when stopping/stopped.

B. Experimental Results

The goal in these experiments is to analyze the behavior of the autonomous vehicle. We present quantitative data followed by an interpretation of the results.

We analyze the behavior by observing the vehicle's speed profile, collision occurrence, and the accumulated rewards for each simulation. To generate the autonomous vehicle's behavior, we first define the input pedestrian behavior. In this experiment we simulate pedestrians moving at constant speed $1.0\,\mathrm{m/s}$, $1.4\,\mathrm{m/s}$, $2.0\,\mathrm{m/s}$ and $2.5\,\mathrm{m/s}$ and also simulate the suicidal (Section VI-A1) and cautious pedestrians (Section VI-A2) as separate experimental conditions.

The trajectories of both pedestrian (red) and autonomous vehicle (blue) is shown in Figs. 5 and 6. The horizontal axis is the time step (in units of 0.2 seconds) and the vertical axis is the distance from the crosswalk (in meters). The slope of the lines represents the velocity. The boxes are a representation of

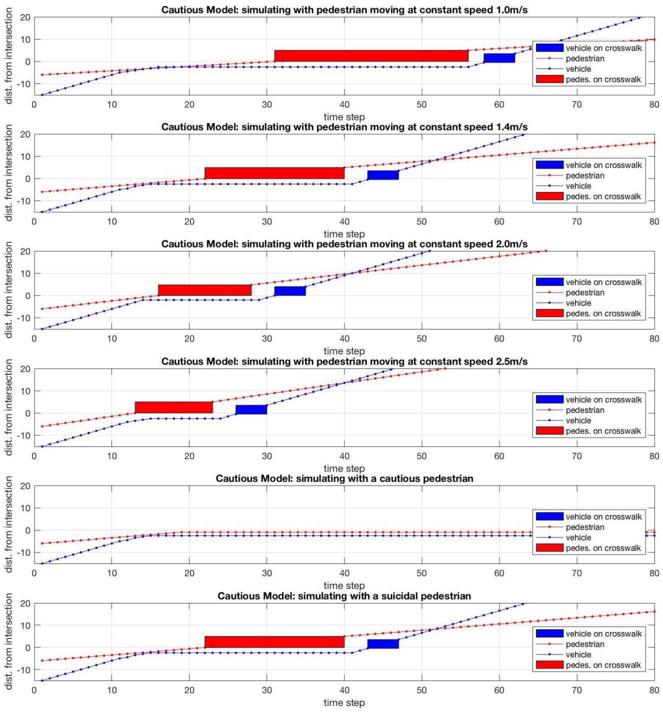

Fig. 6. Cautious pedestrian-based MDP model: We construct a cautious pedestrian-based MDP model and examine the autonomous vehicle's behavior by simulating the model with the same set of experimental conditions as in Fig. 5.

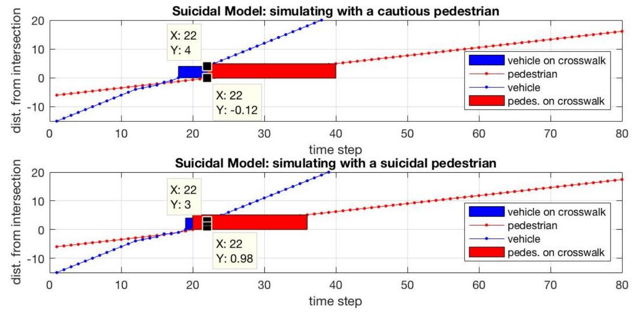

Fig. 7. Suicidal pedestrian-based MDP model with collision penalty as -160: Plot 1 shows that lowering the collision penalty solves the unresolved turn taking issue. However, once the penalty lowers, the modeled vehiclecollides with a suicidal pedestrian, as shown in plot 2.

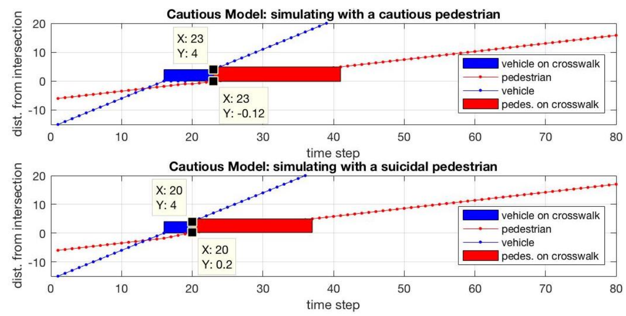

Fig. 8. Cautious pedestrian-based MDP model with collision penalty as -160: The lowered collision penalty solves the unresolved turn taking problem, as shown in plot 1. But lowering the collision penalty also causes the modeled vehicleto collide with a suicidal pedestrian as shown in plot 2.

the crosswalk position. As the crosswalk is $4\,\mathrm{m}$ wide and $5\,\mathrm{m}$ long, the blue and red boxes have different heights.

1) Vehicle's speed profile: The vehicle's characteristics are portrayed directly in its speed profile. The vehicle employing a model of a suicidal pedestrian is more conservative, which is what one would expect: an optimal agent with a pessimistic model must be more risk averse. This is reflected in the vehicle's trajectory— Fig. 5 has shallow slopes, meaning that it slows down over a long period of time, initiating breaking sooner. On the other hand, the vehicle trajectory for the MDP solved under the assumption of a cautious pedestrian has steeper slopes and must perform a hard stop in some cases. For example, the second graph in Fig. 6 shows a rapid stop being executed since the vehicle comes to a complete stop in a short time (around time steps 10–20). The vehicle's model assumes that the pedestrian will be cautious and can thus be counted on to stop before crossing; but the third graph is a pedestrian traveling at constant speed $1.4\,\mathrm{m/s}$ and not stopping, so the vehicle was forced to break hard to avoid a collision.

2) Collision occurrences: The MDP is expected to provide a safe (collision-free) action choice in every state for the autonomous vehicle. We gave a large negative reward R_{col} in order to prevent the pedestrian and vehicle both being on the crosswalk simultaneously. Thus, as the boxes represent the crosswalk area for both pedestrian and vehicle, they should never overlap. By observing the distance between the boxes, we can see how conservative the vehicle is. When we lower the collision penalty, the distance reduces as the vehicle is more willing to risk colliding versus waiting for longer to lower the risk.

3) Unresolved turn taking: The magnitude of the penalty for collisions also affects the possibility of the vehicle and pedestrian approaching the crossing with their respective turn taking still unresolved. In both Fig. 5 and 6, the fifth plot shows an unresolved turn taking situation. The definition of a cautious pedestrian is to wait for the vehicle to cross when it cannot reach the other side whilst simply maintaining the same speed. For instance, the pedestrian in both simulations had stopped to wait for the vehicle to cross first. Whereas, our penalty for collision is large (at -1000) so the vehicle would wait for the pedestrian to cross first to prevent a collision. When the collision penalty is reduced to -160, this deadlock is resolved. The first plots in both Fig. 7 and 8 safely control the vehicle (no boxes overlap) when there is a cautious pedestrian. Unfortunately, lowering the collision penalty now causes the vehicle to be on the crosswalk at the same time (boxes now overlap) as a suicidal pedestrian.

4) Accumulated rewards: Accumulated rewards can be used to compare the performance between different MDP models. Given the same pedestrian behavior as the simulation input, the model with the highest accumulated rewards is most suitable for purposes where the design matches real circumstances. For example, Table I shows that the suicidal pedestrian-based model has a higher reward for simulations

TABLE I
ACCUMULATED REWARDS FOR EACH SIMULATION

	Accumulated Rewards					
	Const. Speed $1.0\,\mathrm{m/s}$	Const. Speed $1.4\,\mathrm{m/s}$	Const. Speed $2.0\,\mathrm{m/s}$	Const. Speed $2.5\,\mathrm{m/s}$	Cautious Pedestrian	Suicidal Pedestrian
Suicidal Pedestrian Based MDP ($R_{col} = -1000$)	165	485	765	865	-695	545
Cautious Pedestrian Based MDP ($R_{col} = -1000$)	900	455	690	795	-725	455

that have an interaction between the vehicle and pedestrian. This result fits with our goal of decreasing the amount of time the vehicle moves slowly. According to our reward definition, every time the vehicle stops, it gets a -10 penalty. Therefore, the simulations that have a shorter vehicle stopping period would have a higher accumulated reward.

VII. CONCLUSION

In this paper, we have proposed an MDP model to explore the how velocity-based signaling affects vehicle-pedestrian interaction in a simple setting under a decision-theoretic model.

We examined our model when there is some divergence between the pedestrian crossing behavior as modeled and as actually occurs. The model shows to result in safe interact when pedestrians behaves similarly to the model assumed within the MDP. The model captures the intuition that planning for treacherous (or adversarial) agents lowers efficiency in more benign circumstances. Rewards are also shown to be a critical part of developing a safe and efficient interaction.

Many avenues are possible for future work. One is to study reward assignments and develop a model that balances efficiency and safety. Another is to study the discretization of the MDP model states: increasing the resolution of the states may increase performance at the cost of memory efficiency. Nonuniform discretizations may save representational cost but will lose some information. Some means for balancing the two would be ideal.

VIII. DISCLAIMER

IX. ACKNOWLEDGMENT

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