A new model for self-organized robotic clustering: understanding boundary induced densities and cluster compactness

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Abstract— **For self-organized multi-robot systems, one of the widely studied task domains is object clustering, which involves gathering randomly scattered objects into a few piles. Earlier studies have pointed out that environmental boundaries influence the cluster formation process, generally causing clusters to form around the perimeter rather than centrally within the workspace. But it is usually central clusters that are desired in robotic clustering systems. In this paper, we derive general conditions that prevent the problem of boundaries causing perimeter clusters. We develop a mathematical model to explain how sets of clusters evolve into a single cluster without any boundary cluster being formed. Through analysis of the model, we show that time-averaged spatial densities of the robots play a significant role in producing conditions that ensure a single central cluster emerges. Thus, local densities of robots can be considered a system-level control parameter to achieve this task. We further investigate how the physical packing of clusters affects clustering dynamics. To do this, we introduce a measure of scaled compactness and show that the lifetime of clusters is well predicted by this descriptor.**

I. Introduction

Studies of self-organized multi-robot systems attempt to understand how to coordinate systems composed of large numbers of simple robots, each typically have limited sensing, communication, and computational capabilities. Despite these minimal capabilities, self-organized multi-robot systems have been shown to exhibit complex collective behaviors via a combination of positive and negative feedback and stigmergy. For these systems, the fundamental research challenge is to tame the problem of emergent complexity and turn it toward engineering ends. A crucial ingredient in this is building models to understand self-organization better.

Object clustering was widely studied in the early days of biologically-inspired multi-robot research. The task of object clustering involves gathering scattered objects into a single pile. Practical uses for this behavior are scarce, but some authors propose employing clustering within a broader pipeline of activities, much like raking leaves into a single pile simplifies their subsequent processing, or in preparing construction sites [1]. Within the existing works, the robots typically execute a very simple control strategy. The strategy usually amounts to a random walk in the workspace, with the sensor(s) indicating when some number of objects are encountered. This eventuality triggers some action, usually a turn, before the robots resume their random walk. It is remarkable that such a simple policy forms clusters repeatedly and reliably. Several authors have provided explanations for how clusters emerge through this process. Ultimately, all these explanations boil down to a geometric argument which says that the shape of the clusters either (1) causes greater accretion with increasing size, or (2) less attrition with increasing size, or (3) both.

While classic works focused on cylindrical objects (*e.g.*, pucks, frisbees), our work has been exploring the task with square objects. For a problem where the conventional wisdom that explains the clustering process hinges on geometric arguments, it is somewhat surprising that no consideration has been given to varying the shape of the objects. Square objects heighten two effects which have been given a cursory treatment in the existent work. Perhaps rather obvious is the fact that square objects which get pushed against a boundary usually remain there, producing cluster growth at the boundaries rather than in the central region. Our prior work explored methods to ameliorate this effect by employing heterogeneous strategies by robots. Reflection on why this solution worked lead to the realization that we were manipulating the densities of robots so that impulses related to box configurations near the boundary were increased.

The second observation is that existing models assume a cluster of n items is symmetric and well-packed. Moving beyond cylindrical objects, the shortcomings of this simplification become acute. There can be a vast number of variables in the stability of different arrangements of boxes, and even the most stable configuration is not a smooth function of n.

As a consequence of both these observations, we enrich prior models by treating spatially the densities of robots, specifically with respect to a difference between boundaries and the center of the workspace, and propose a model for cluster compactness.

II. Related Work

Deneubourg *et al.* [2] first proposed a sorting algorithm inspired by brood sorting of ants and demonstrated how their behavior could be used in a simulated multi-agent system. Their simple behaviors, with only a local density sensor and without direct communication between robots, successfully achieved sorting. Thereafter, Beckers *et al.* [3] demonstrated clustering behavior without requiring a density sensor. The only sensor they used was a binary threshold on force. They also explain the emergence of clusters via a probabilistic analysis based on geometric characteristics of the clusters.

In a mathematical analysis of clustering dynamics, Martinoli [4] introduced a probabilistic model to quantify the geometric notion under the assumption of rotationally symmetric clusters and demonstrated it through simulations as

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well as physical experiments. Kazadi *et al.* [5] also proposed a mathematical model of clustering dynamics, describing cluster growth properties and arguing along similar geometric lines to Martinoli.

Most previous work has indicated that environmental boundaries affect the clustering process [6], [7]. For example, Maris and Boeckhorst [6] considered objects to be lost once they were pushed against the boundary. Using square objects makes the task rather challenging because flat edges exacerbate adhesion to the boundary. Once against the boundary, it is difficult for a cylindrical robot to move a box into the center of the environment. This can be observed in the video posted by Vaughan [8] in which 36 iRobot Creates created clusters of square objects running only their default program.

However, despite the significant influence of the boundary on the clustering process, many authors focus on empirical demonstrations and ignore environmental effects like those caused by the boundary in their models.

III. Extending Analysis of Clustering Systems: A Matter of the Boundary

Earlier studies show that clustering along the boundary of the workplace harms central clustering performance. Nonetheless, many studies still ignore or simplify the effect of the boundary when building a model. One standard way is to ignore where the clusters are formed. This is fallacious because the boundaries themselves buttress clusters, making them behave as if they are part of a much larger cluster. Next, we introduce an extended analysis of the clustering system by considering the effect of the boundary and derive a condition that is required to avoid the boundary interference.

A. Review: A Novel Approach for Object Clustering

In our previous work, we introduced two behaviors (*twisting* and *digging*) where the robots in a team were assigned to one of these behaviors with all robots executing their behavior concurrently. We demonstrated that groups with a mix of robots operating either of the two complementary behaviors successfully generated a single central cluster and overcame the problem of boundary cluster formation (see [9], [10], [11] for more detail).

Fig. 1 depicts scenarios possible with the robot behaviors during a clustering process. Twisters are likely to push objects against the boundary or bring objects into the center. On the other hand, diggers further separate twisted objects from the boundary, but there is comparatively little chance that twisted objects will be brought into the center because those robots stay near the boundary. We simplify this by assuming that diggers only interact with boundary objects by performing the prying action, whereas twisters interact with all objects by randomly moving in the entire workplace. Fig. 2 provides the evidence that supports this idea by showing a spatial distribution of a twister and a digger.

In general, clusters undergo accretion or attrition through the random interactions between robots and objects (or object clusters). That is, if the number of deposited objects in any cluster exceeds the number of removed objects, the cluster has an accretive tendency. Whereas the cluster has an

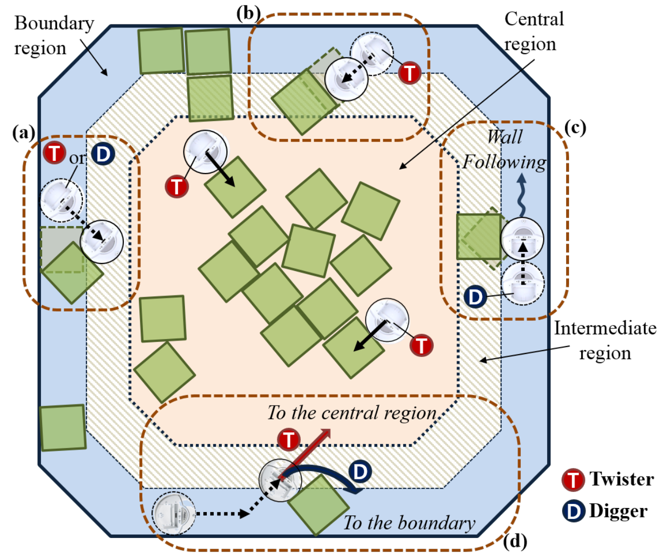

Fig. 1. A pictorial representation of the clustering process. (a) Prying objects away from the boundary. Both the twister and the digger can pry a square object loose from the boundary by hitting the object's corner. (b) Twisting behavior. A twister pushes the box shifted by the prying motion and brings it into the center. (c) Digging behavior. A digger forms gaps between boxes and boundaries and prevent boundary cluster growth. (d) Trajectories of twisters and diggers after the prying motion. Twisters move into the center while diggers proceed along a curved path to detect the boundary.

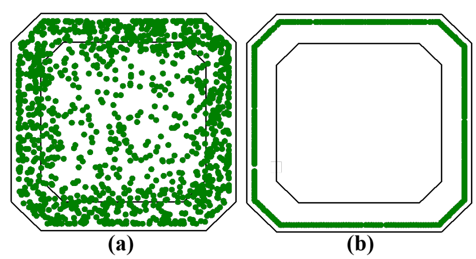

Fig. 2. Spatial distribution of (a) a twister and (b) a digger. 1200 positions of a single twister and a single digger are plotted in the workplace, without any objects, every second. In the ideal case without interactions with other objects, the twister covers most areas of the workplace, while the digger moves around the perimeter of the boundary.

attritional tendency if deposits are smaller than removals. If both are the same, the cluster is in equilibrium. Based on this theoretical analysis, Kazadi *et al.* [5] proposed a clustering model through a characteristic function that represents the growth properties of the clusters. However, this model is also built without considering the effect of the boundary, the positions of the robots, or even the positions of the objects.

In order to analyze the clustering system with consideration of the boundary interference, we divide the workplace into a central region and a boundary. Through interactions between robots and objects in the workplace, multiple clusters emerge in the central region or the boundary. We expand the clustering model to consider the effects of the boundary.

B. An Extended Model of the Clustering System

1) A model for the boundary: If multiple robots possess a special treatment to release stuck boxes from the boundary we can model a transition back toward the central region.

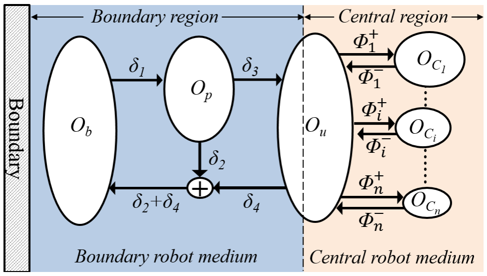

Fig. 3. Abstract state diagram of the proposed clustering system.

(If not, this flux will work out to be zero, as expected.) We consider a robotic clustering system that is composed of the two behaviors designed in our prior work [9], [10].

Fig. 3 shows the abstract state diagram of our clustering scenario. The robots are capable of pushing an object and depositing or removing an object from any cluster. This means the robots can be thought of as a medium in which the clusters occur. We consider one form of robot media for the boundary and another for the central region. We also assume that objects belong to one of four possible states: the boundary objects (O_b), the objects pried away from the boundary (O_p), the unclustered objects in the central region (O_u), and the objects belonging to any central cluster (O_{C_i}).

Let us first examine the robot medium and objects in the boundary region (we will investigate the central region in the next section). While clustering work proceeds, the unclustered objects in the central region O_u can be transferred to the boundary O_b through the boundary robot medium. On the other hand, objects in O_b may still stick on the boundary or be detached from the boundary by robots' special treatment, like a prying motion in our clustering system. We define O_p as an intermediate transition state (transition from O_b to O_u), which represents objects pried away from the boundary. The objects in O_p can be transferred to O_u or revert to O_b by interactions with robots moving into the boundary.

Let δ be the number of objects transferred between states during a time interval. According to the diagram in Fig. 3, the rates of change of objects in the boundary region will be

$$\frac{dO_u}{dt} = \delta_3 - \delta_4, \tag{1}$$

$$\frac{dO_p}{dt} = \delta_1 - \delta_2 - \delta_3, \tag{2}$$

$$\frac{dO_b}{dt} = \delta_2 + \delta_4 - \delta_1. \tag{3}$$

In general, self-organized clustering progress is non-stationary because robots randomly interact with objects. On the basis of this characteristic, we assume that the rate of change of objects in each state depends on the averaged frequency of interactions between individual robots and objects in the same region. That is, the more frequent contacts between objects and the robots available to manipulate them, the greater the number of the state transitions of objects. Then δ can be expressed in terms of the local densities of robots available to transfer an object and the likelihood of object removal in any state. Let $\rho^{\circ}_{T(\cdot)}$ and $\rho^{\circ}_{D(\cdot)}$ be the time-averaged local densities of twisters and diggers capable of removing objects in a given state. Also, let $L^-_T(\cdot)$ and $L^-_D(\cdot)$ be the likelihood of object removal in a certain state by twisters and diggers, respectively. In the previous section, we also explained that diggers only interact with boundary objects O_b, whereas twisters interact with all objects in the entire workplace. Accordingly, each δ can be written as

$$\delta_1 = \rho^{\circ}_{D(O_b)} \cdot L^-_D(O_b), \tag{4}$$

$$\delta_2 = \rho^{\circ}_{T(O_p \to O_b)} \cdot L^-_T(O_p), \tag{5}$$

$$\delta_3 = \rho^{\circ}_{T(O_p \to O_u)} \cdot L^-_T(O_p), \tag{6}$$

$$\delta_4 = \rho^{\circ}_{T(O_u \to O_b)} \cdot L^-_T(O_u). \tag{7}$$

2) The synthesis of a general model: We developed an extended model that reflects clustering dynamics in the boundary area. With this extension, we explain how general clustering systems work, including the effect of the boundary.

Now suppose the entire workplace is partitioned into a central region and a boundary region. The clustering process in the central region can be treated with models such as those by [5] or [12]. As shown in Fig. 3, multiple central clusters interact with unclustered objects through the central robot medium. By picking subscripts, without loss of generally, we assume that the sizes of central clusters are $C_1 > C_i > C_n$. That is, C_1 is the largest central cluster and C_n is the smallest. The rate of change of the i^{th} central cluster will be

$$\frac{dO_{C_i}}{dt} = \Phi_i^+ - \Phi_i^-, \tag{8}$$

where Φ_i^+ is the number of unclustered objects that are deposited into the i^{th} central cluster and Φ_i^- is the number of objects that are removed from the i^{th} central cluster. According to a probabilistic analysis based on geometric characteristics of clusters, larger clusters will be more likely to obtain objects and less likely to lose objects than smaller clusters.[1] This means $\frac{dO_{C_i}}{dt} = \Phi_i^+ - \Phi_i^- > 0$ and $\frac{dO_{C_n}}{dt} = \Phi_n^+ - \Phi_n^- < 0$. Under a recurrence of this clustering process, the large clusters grow bigger, while the smallest cluster becomes smaller and will eventually disappear. The model for central clusters will also be extended by the notion of the compactness of the clusters in the next section.

By considering clustering dynamics not only in the central region but also in boundary clusters, we can more precisely understand the evolution of the largest central cluster, and the decay of boundary clusters.

C. Conditions to Prevent Boundary Cluster Growth

We have developed rate equations to describe the extended clustering system. We now turn to examine a condition to prevent boundary clusters from growing.

A net flow of objects between the boundary and the central region is determined by the interactions between the boundary robot medium and O_u. That is, if $\frac{dO_u}{dt} = \delta_3 - \delta_4 > 0$ in the boundary region, the number of objects in O_b will decrease and the quantity of O_u will increase as time progresses. If we assume that the quantity of the intermediate state O_p is stationary, we obtain $\frac{dO_p}{dt} = \delta_1 - \delta_2 - \delta_3 = 0$. Rearranging $\delta_3 - \delta_4 > 0$ and $\delta_1 - \delta_2 - \delta_3 = 0$, we can find that

[1]The basic argument is that it is easier to strike a smaller cluster at an oblique angle that draws away an object than a large one. A very large cluster has only tangents to the cluster perimeter.

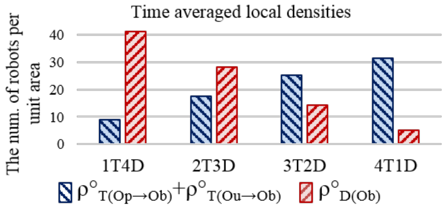

Fig. 4. Time averaged local densities with respect to the ratio of twisters to diggers.

$\delta_3 > \delta_4$ and $\delta_3 = \delta_1 - \delta_2$. Thus, a condition required for O_u to grow and for O_b to shrink may be written as $\delta_1 - \delta_2 > \delta_4$ and so

$$\delta_2 + \delta_4 < \delta_1. \tag{9}$$

From (4), (5), (7), and (9), in order to prevent boundary cluster growth, we can obtain

$$\rho^{\circ}_{T(O_p \to O_b)} \cdot L^{-}_{T}(O_p) + \rho^{\circ}_{T(O_u \to O_b)} \cdot L^{-}_{T}(O_u) < \rho^{\circ}_{D(O_b)} \cdot L^{-}_{D}(O_b). \tag{10}$$

Since the majority of diggers that encounter boundary objects successfully pry them away from the boundary in our system, we assume that $L^{-}_{D}(O_b) \simeq 1$. Furthermore, because twisters can always push a single object (*e.g.*, an unclustered or a pried object without loss), we assume that $L^{-}_{T}(O_p) \simeq 1$ and $L^{-}_{T}(O_u) \simeq 1$. Under these assumptions, in order for boundary objects to be removed from the boundary region,

$$\rho^{\circ}_{T(O_p \to O_b)} + \rho^{\circ}_{T(O_u \to O_b)} < \rho^{\circ}_{D(O_b)}. \tag{11}$$

We hypothesize, if the time averaged local densities of twisters and diggers satisfy (11), the boundary interference in the clustering system will be negligible or eliminated entirely.

D. Experimental Results

In order to test how the local densities of the robots influence the formation and removal of boundary objects, we implemented a simulator for robotic clustering systems with Box2D. Box2D is a 2D physics engine and an open source C++ engine for simulating rigid bodies [13]. The simulation environment is designed to the scale of the real environment in our prior work [10]. We performed simulations for all mixes of twisters (T) and diggers (D) with 5 robots: 1T4D, 2T3D, 3T2D, and 4T1D. We also used 20 objects, and 20 runs, each lasting 20 minutes (simulation speed is six times faster than the physical experiment).

We obtain the local densities of available robots capable of manipulating objects, $\rho^{\circ}_{T(O_p \to O_b)}$, $\rho^{\circ}_{T(O_u \to O_b)}$, and $\rho^{\circ}_{D(O_b)}$, which only affect the condition (11). To measure the sum of $\rho^{\circ}_{T(O_p \to O_b)}$ and $\rho^{\circ}_{T(O_u \to O_b)}$, we observe the frequency of twisters moving from the central region to the boundary through the intermediate region during all trials. Then, we obtain the sum of $\rho^{\circ}_{T(O_p \to O_b)}$ and $\rho^{\circ}_{T(O_u \to O_b)}$ by dividing the frequency by the perimeter of the central region. $\rho^{\circ}_{D(O_b)}$ is measured by counting the number of diggers passing through a point on the boundary during the same period.

Fig. 4 shows the time averaged local densities for the particular ratio of twisters to diggers in the simulation. According to our hypothesis, 1T4D and 2T3D satisfy the condition in (11) to stop boundary cluster growth. In order to validate our hypothesis, we analyzed the frequency of boundary objects over time. Fig. 5 presents the number of boundary objects with respect to the labor mix during the clustering process in the simulation. As hypothesized, all trials for 1T4D and 2T3D successfully remove boundary objects. In contrast, the robots of 3T2D and 4T1D failed to eliminate boundary objects and even made the boundary clusters grow. Table I also shows the averaged object distribution of 20 trials in each mixed strategy. We can observe that since the average size of O_b in 1T4D and 2T3D is below 1, those strategies can prevent forming boundary clusters. However, 3T2D and 4T1D struggled to eliminate the boundary objects. That is, the simulation results support our hypothesis.

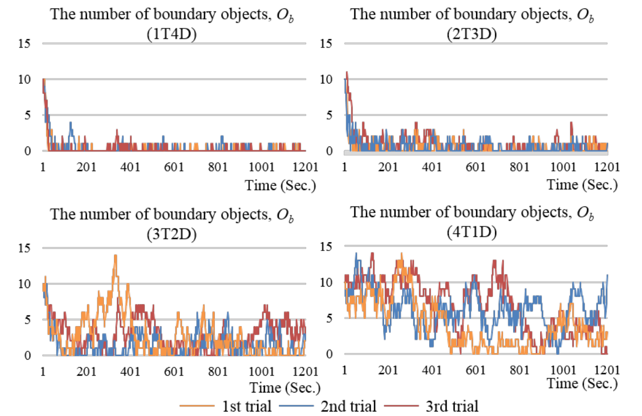

Fig. 5. The number of boundary objects over time during the clustering process. Observed data of three of 20 trials for each labor mix are plotted.

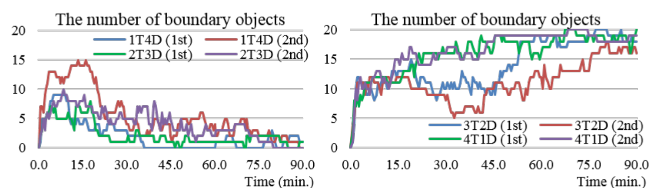

Fig. 6. Boundary objects observed over time in physical experiments

TABLE I

AVERAGED OBJECT DISTRIBUTION.

	The number of objects		
	O_u	O_p	O_b
1T4D	18.345	1.3708	0.3006
2T3D	17.712	1.6706	0.6344
3T2D	15.431	2.2206	2.3647
4T1D	11.995	2.3119	5.7097

In physical experiments, the results are similar to the simulation results. Each trial lasted 90 minutes, with 20 objects. As shown in Fig. 6, all trials of 1T4D and 2T3D successfully prevented forming boundary clusters. However, 3T2D and 4T1D failed to remove objects from the boundary and eventually formed only boundary clusters.

Fig. 7 and Fig. 8 show the configurations in the simulation and the physical experiments. We observed in both environments that 1T4D successfully formed a single central cluster, but 4T1D failed throughout. In short, if we can control the local densities of robots having a different purpose (here, twisting and digging behaviors), we can prevent the boundary interference in the clustering system which harms central clustering performance. We are not aware of other

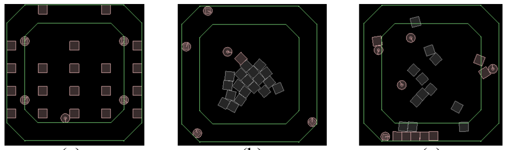
(a) (b) (c)

Fig. 7. Simulation experiments. (a) Initial configuration. (b) Final configuration (1T4D). (c) Final configuration (4T1D).


(a) (b) (c)

Fig. 8. Physical experiments. (a) Initial configuration. (b) Final configuration (1T4D). (c) Final configuration (4T1D).

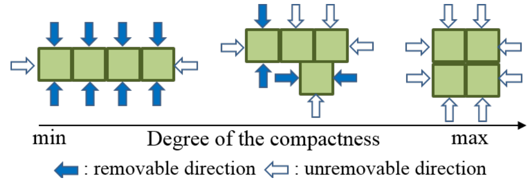

Fig. 9. Differing compactness for clusters of the same size ($n = 4$). The more compact cluster is robust because the directions of contacts that the robot can remove the object are few.

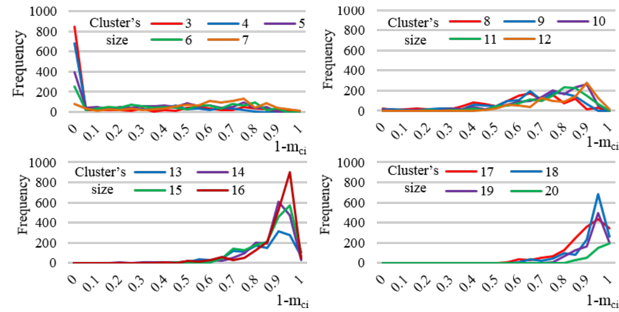

Fig. 10. Frequency of clusters occurring by compactness measure, which is not scaled by the cluster's size. The horizontal axis describes the normalized compactness according to clusters' size.

work which has influenced the density of robots in order to influence cluster formation, let alone in a systematic way.

IV. Extended Analysis of Clustering Systems: A Compactness of Clusters

In this section, we extend analysis by investigating the role that the compactness of clusters has on the clustering process. As mentioned in Section II, prior studies have developed theories to describe how clustering systems work, however, the theories introduce simplifying assumptions. A common assumption is that the geometry of a cluster is rotationally symmetric. However, since generally the clusters formed in practice are asymmetric, we need to closely examine the shape of the clusters for more accurate analysis.

A. The Compactness of Clusters

The compactness of a cluster reflects the degree of dispersion of items packed within the cluster. Fig. 9 shows that the shape of a highly compact cluster of square items is close to the square symmetry whereas a less compact cluster is line-shaped. Moreover, for a given number of items, the compactness relates to the number of ways items can be removed from the cluster. Thus, a quantitative measure of the compactness is expected to capture information about the geometric configuration of a cluster and its robustness.

To quantify the degree of compactness, we compute the second moment indicating a geometrical property of an area. In the penny-packing problem, Graham and Sloane proposed that minimizing the second moment of the pennies as a solution for the tightest packing [14]. The second moment of cluster c_i, $m_{c_i} = \frac{1}{d^2}\sum_{j=1}^{n}\left\|p_{c_{ij}} - \bar{p_{c_{ij}}}\right\|^2$, where, d is the width of the object, n is the size of the cluster c_i, $p_{c_{ij}}$ is the center point of the i^{th} object in cluster c_i, and $\bar{p_{c_i}}$ is $\frac{\sum p_{c_{ij}}}{n}$.

A cluster that has a minimal moment is a compact cluster, as all of the objects are distributed as closely to the cluster's centroid as possible. However, since the range of the cluster's second moment depends on its size, we normalize and scale it according to the cluster's size. Here, we can define the compactness of a cluster as

$$\Gamma_{c_i} = (1 - \tilde{m_{c_i}}) \times n_{c_i}, \tag{12}$$

where, $\tilde{m_{c_i}}$ is the second moment normalized to lie in [0,1], n_{c_i} is the number of objects in the cluster c_i.

From (12), the compactness of a line-shaped cluster will be computed as the minimum value (it has the maximum second moment). This means the objects in the line-shaped cluster will be easily removed by the robots. Whereas in square symmetric clusters it is just the opposite (See Fig. 9). Therefore, we can see that although the clusters' sizes are the same, the likelihood of growing the clusters might not correspond according to the compactness of clusters. In other words, the compactness is an essential factor to consider when understanding how clustering occurs. Since we consider an extended clustering system including the analysis of boundary clusters, we distinguish between the compactness of central clusters and boundary clusters by multiplying -1 for the boundary case. Thus,

$$\Gamma_{cc_i} = (1 - \tilde{m_{cc_i}}) \times n_{cc_i},\ \Gamma_{bc_j} = -(1 - \tilde{m_{bc_j}}) \times n_{bc_j}, \tag{13}$$

where, cc_i is the i^{th} central cluster and bc_j is the j^{th} boundary cluster. These signed measures will be useful for analyzing precisely the clustering process, as we show next.

B. The Analysis of the Impact of the Compactness on the Clustering Process

In order to investigate the impact of compactness on clustering dynamics, we measured the frequency of the live clusters according to their compactness for a fixed size of clusters and we compare this across cluster sizes. We define the size of a cluster as a group of more than tree objects, each contacting at least one other. In Fig. 10, we show the result as partitioned in three regimes, depending on the size: small (3–6 objects), middle (7–13 objects), and large (14–20 objects) size of clusters. When clusters are small, they are easily broken by robots unless the compactness is increased sufficiently. The small cluster frequencies are relatively high at low compactness because the clusters repeat the cycle of creation and extinction, *i.e.*, churn. For middle-sized clusters, increasing the compactness for a given size causes a growth

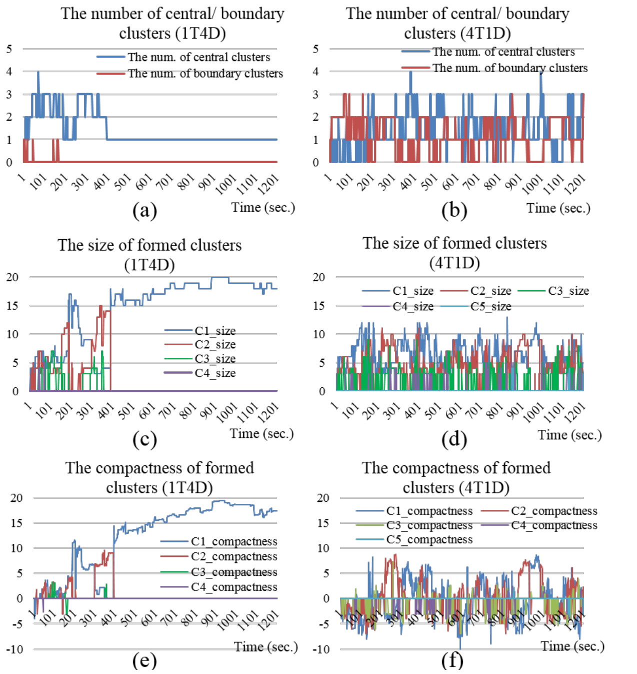

Fig. 11. Comparison between (a), (c), (e) successful 1T4D and (b), (d), (f) failed 4T1D in the simulation.

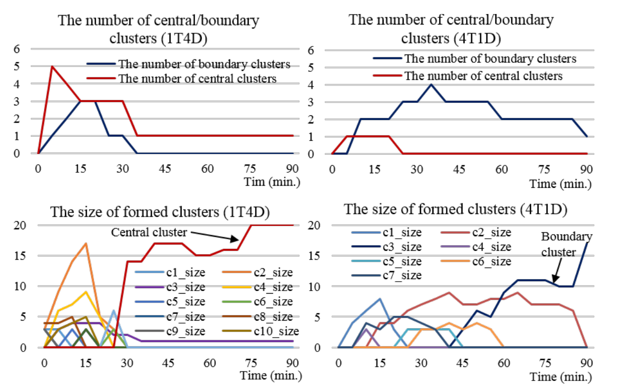

Fig. 12. Comparison of results between successful 1T4D and failed 4T1D in physical experiments. We observed configurations at intervals of 5 mins.

point. For this reason, the frequencies of middle clusters are slowly increased and have relatively low frequencies overall, *i.e.*, they represent a transition zone. The large clusters have a comparatively long lifetimes with large compactness. That is, the large clusters, which are packed tightly are robust against removal actions of the robots. For example, when the cluster's size is 16, the cluster has a long life because the cluster has the maximum compactness (square shape of 4×4). This observation supports the suggestion that cluster compactness is a significant factor.

To understand the clustering process with the compactness, we compared the results of a successful experiment (1T4D) and a failed experiment (4T1D). As shown in Fig. 11 (a) and (c), even though 1T4D repeatedly created and destroyed small central and boundary clusters at the beginning, a single central cluster is finally formed after 420 sec. This result can be explained by the compactness of the largest cluster. In Fig. 11 (e), we observe that if the compactness of the largest reaches a threshold level (here, 15), and it is a stable and robust cluster. That is, the largest central cluster remains steady by consistently increasing the compactness. On the other hand, although 4T1D tried to generate central clusters, the central clusters could not evolve into a larger cluster due to the boundary interference and the lack of its compactness (See Fig. 11 (b), (d) and (f)). As shown in Fig. 12, similar trends are observable in physical experiments.

V. Conclusion

We proposed a model to help understand the dynamics of a multi-robot clustering system. We contribute a model which considers heterogeneity in different behavior as a function of location. Through this we can capture a notion of local spatial density, and also model state transitions in the object being clustered (in this work, a transition of the object into a pried mode), and context dependency (boundary objects are modeled as behaving differently from one in the central region). Using this model, we derived conditions for evolution of the largest central cluster and for degeneration of boundary clusters. We also investigated an important factor which has been overlooked in the literate: there can be a significant difference between two clusters of the same size. We construct a measure, "scaled compactness" which characterizes this fact, showing that it helps understand three different regimes of cluster evolution.

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